FOURIER TRANSFORMS AND SIGNAL PROCESSING

Due date: 04/17/2024

Fourier transforms are one of the most commonly used signal processing tools. In a nutshell, think of a time-domain function $h(t)$ and a corresponding frequency-domain function $H(f)$ as two representations of a single function, with the following mapping that lets you go back and forth between the two:

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{i2\pi f t} dt, \quad h(t) = \int_{-\infty}^{\infty} H(f) e^{-i2\pi f t} df.$$

For N discrete data points $h(t_k) \equiv h_k$, the discrete Fourier transform is given by:

$$H(f_n) \equiv H_n = \sum_{k=0}^{N-1} h_k e^{i2\pi kn/N},$$

and its corresponding inverse is given by:

$$h(t_k) \equiv h_k = \frac{1}{N} \sum_{n=0}^{N-1} H_n e^{-i2\pi kn/N}.$$

Of course, as we discussed in class, no one in their right mind would ever do DFT instead of FFT if they can help it.

a) Compute and plot the Fourier transform and the inverse Fourier transform for the following functions: sine, cosine, square pulse, triangle, delta. Compute the (one-sided) power spectral density for all functions. This should give you a clear idea of the Fourier transform.

b) Cook up a mixture of sines and cosines, i.e. $h(t) = \sin \pi t + 3 \sin 3\pi t + 5 \cos 5\pi t$. Try the same with non-integer frequencies. Demonstrate aliasing and verify that spectral power is conserved.

c) Analyze the sound of a guitar (the wav and the text files are on the course homepage).

d) Listen to Bach's Partita in the uncompressed mp3 format. Then listen to a 2.3 s exerpt that has been sampled at different rates: 882, 1378, 2756, 5512, 11025 and 44100. Once you hear what the effect of undersampling is, quantify it by the Fourier transform. All data (in both mp3 and ascii) are available on the course webpage.

e) Compute the autocorrelation function of the sound of boiling water. If you are so inclined, acquire your own data, otherwise you can use `boiling.data` from the course homepage. *Hint:* the autocorrelation function will likely drop rapidly and you should use the log scale to visualize it.

f) Using convolution, predict the shape of a spectral line out of a diffraction-limited spectrograph. *Hint:* show (or assume) that the response function is a gaussian.